Sample Question Paper - 29
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Find the roots of the quadratic equation $x^2 + 4\sqrt{2}x + 6 = 0$.

2. If the distance between two parallel tangents drawn to a circle is 20 cm, then find the radius of the circle.

3. Find the discriminant of the quadratic equation $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$.

OR

The sum of the squares of two consecutive natural numbers is 41. Represent this situation in the form of a quadratic equation.

4. If $d_i = x_i - 25$, $\Sigma f_i d_i = 200$, $\Sigma f_i = 100$, then find the mean ($\bar{x}$).

5. If 7 is the common difference of an A.P., whose k^{th} term is a_k. Find $a_{k+1} - a_k$. If $a_9 = 16$, then find a_{10}.

6. If the areas of three adjacent faces of a cuboid are x, y and z respectively, then find the volume of the cuboid is the terms of xyz.

OR

The material of a cone is converted into the shape of a cylinder of equal radius. If height of the cylinder is 8 cm, then find the height of the cone.

SECTION - B

7. Find the mode of the given data.

Class interval	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Frequency	15	37	56	87	115

8. If the length of shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Then find the angle of elevation of the sun.

OR

If the angle of elevation of the sun is 30°, then find the length of the shadow cast by a tower of 150 feet height. And, if the angle of elevation changes to 45° from 30°, then find the relation between the length of tower and its shadow.

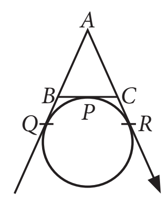
9. Draw a line segment of length 3.3 cm and divide it in the ratio 3 : 8. Measure the two parts.

10. Find the median for the following data.

Class interval	20–40	40–60	60–80	80–100
Frequency	10	12	20	22

SECTION - C

11. A circle touches the side BC of a ΔABC at P and AB and AC when produced touches the circle at Q and R, respectively as shown in the figure. Show that $AQ = \frac{1}{2}$ (Perimeter of ΔABC).



OR

A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

12. A manufacturer of laptop produced 6000 units in 3rd year and 7000 units in the 7th year. Assuming that production increases uniformly by a fixed number every year, find

(i) the production in the 1st year,

(ii) the production in the 5th year,

(iii) the production in 6th year.

Case Study - 1

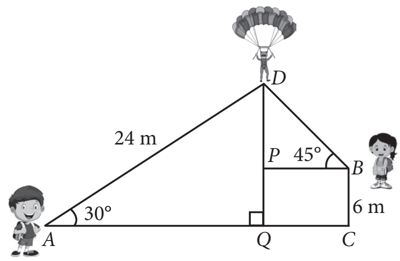
13. Suraj took 4 small spherical balls of silver of surface area 887.04 sq.cm each from a blacksmith. He wanted them to be made into cylindrical coins of radius one-fourth of that of the silver ball and height 4 cm.

(i) Find the radius of each spherical ball.

(ii) Find the curved surface area of each coin.

Case Study - 2

14. Karan and his sister Riddhima visited at their uncle's place-Bir, Himachal Pradesh. During day time Karan, who is standing on the ground spots a paraglider at a distance of 24 m from him at an elevation of 30°. His sister Riddhima is also standing on the roof of a 6 m high building, observes the elevation of the same paraglider as 45°. Karan and Riddhima are on the opposite sides of the paraglider.



Based on the above information, answer the following questions.

(i) Find the distance of paraglider from the ground.

(ii) Find the distance between the paraglider and the Riddhima.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. We have, $x^2 + 4\sqrt{2}x + 6 = 0$

$\Rightarrow \quad x^2 + 3\sqrt{2}\,x + \sqrt{2}\,x + 6 = 0$

$\Rightarrow \quad x(x + 3\sqrt{2}) + \sqrt{2}\,(x + 3\sqrt{2}) = 0$

$\Rightarrow \quad (x + \sqrt{2})(x + 3\sqrt{2}) = 0$

$\Rightarrow x = -\sqrt{2}$ or $x = -3\sqrt{2}$

$\therefore$ Roots are $-\sqrt{2}$ and $-3\sqrt{2}$.

2. Distance between two parallel tangents drawn to a circle is the diameter of the circle.

So, radius of the circle $= \frac{20}{2} = 10$ cm.

3. The given equation is $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$.

Here, $a = \sqrt{3}$, $b = -2\sqrt{2}$ and $c = -2\sqrt{3}$

Now, $D = b^2 - 4ac$

$\Rightarrow D = (-2\sqrt{2})^2 - 4 \times \sqrt{3} \times (-2\sqrt{3}) = 8 + 24 = 32$

OR

Let the two consecutive natural numbers be x and $x + 1$. Then, their squares are x^2 and $(x+1)^2$ respectively.

$\therefore$ The required equation is, $x^2 + (x + 1)^2 = 41$

$\Rightarrow x^2 + x^2 + 1 + 2x = 41$

$\Rightarrow 2x^2 + 2x - 40 = 0 \Rightarrow x^2 + x - 20 = 0$

4. Here, $d_i = x_i - 25 \Rightarrow a = 25$.

Also, $\sum f_i d_i = 200$ and $\sum f_i = 100$ $[\because\ d_i = x_i - a]$

Now, mean, $\bar{x} = a + \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right) = 25 + \frac{200}{100} = 27$

5. a_k and a_{k+1} are successive terms of given A.P.

$\therefore \quad a_{k+1} - a_k = d \Rightarrow a_{k+1} - a_k = 7.$

Now, $a_9 = 16$

$\therefore \quad a_{10} - a_9 = 7$

$a_{10} = 7 + 16 = 23$

6. Let l, b and h respectively be the length, breadth and height of the cuboid.

Since x, y and z are the areas of three adjacent faces of cuboid.

$\therefore \quad x = lb, y = bh, z = hl$

Now, $xyz = (lb)(bh)(hl) = (lbh)^2 \Rightarrow lbh = \sqrt{xyz}$

Hence, volume of the cuboid $= \sqrt{xyz}$

OR

Let r be the radius of cone & cylinder and h be the height of cone.

$\because$ Volume of cone = Volume of cylinder

$\therefore \quad \frac{1}{3}\pi r^2 h = \pi r^2(8) \Rightarrow h = 3 \times 8 = 24$ cm

7. The frequency distribution table from the given data can be drawn as :

Class interval	Frequency
0-20	15
20-40	37 – 15 = 22
40-60	56 – 37 = 19
60-80	87 – 56 = 31
80-100	115 – 87 = 28

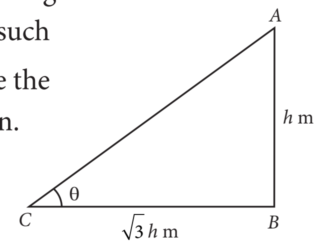
The modal class is 60–80 as it has the maximum frequency.

$\therefore \quad l = 60, f_0 = 19, f_1 = 31, f_2 = 28$ and $h = 20$

$$\Rightarrow \quad \text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{31 - 19}{2 \times 21 - 19 - 28}\right) \times 20 = 76$$

8. Let AB be the tower of height h m and BC be the shadow of the tower, such that $BC = \sqrt{3}\,h$ m. Let θ be the angle of elevation of the sun.



In ΔABC,

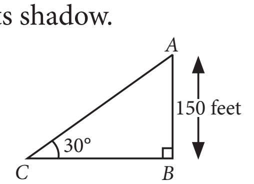
$\tan\theta = \frac{AB}{BC} = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \tan 30° \Rightarrow \theta = 30°$

Hence, angle of elevation of the sun is 30°.

OR

Let AB be the tower and BC be its shadow.



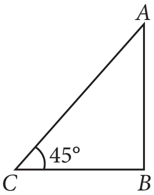
In ΔABC, $\tan 30° = \frac{AB}{BC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{150}{BC}$

$\Rightarrow \quad BC = 150\sqrt{3}$ feet

Hence, length of shadow of tower is $150\sqrt{3}$ feet.

Now, angle of elevation $= 45°$



$\therefore$ In ΔABC, $\tan 45° = \dfrac{AB}{BC}$

$\Rightarrow 1 = \dfrac{AB}{BC} \Rightarrow AB = BC$

$\therefore$ Length of tower and length of its shadow are equal.

9. Steps of construction :

Step-I : Draw a line segment $AB = 3.3$ cm.

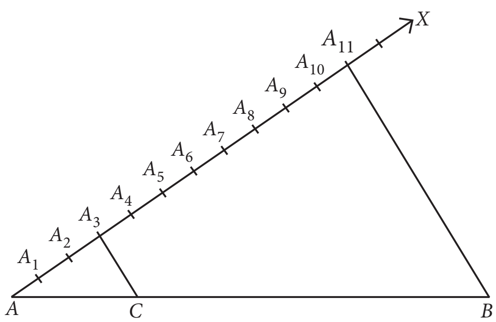
Step-II : Draw any ray AX, making an acute angle with AB.

Step-III : Locate 11 (= 3 + 8) points $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}$ and A_{11} on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = A_7A_8 = A_8A_9 = A_9A_{10} = A_{10}A_{11}$

Step-IV : Join BA_{11}.

Step-V : Through the point A_3, draw a line parallel to $A_{11}B$ intersecting AB at the point C.

Then, $AC : CB = 3 : 8$



By measurement, we have

$AC = 0.9$ cm and $CB = 2.4$ cm.

10. The cumulative frequency distribution table from the given data can be drawn as :

Class interval	**Frequency (f_i)**	**Cumulative frequency (*c.f.*)**
20-40	10	10
40-60	12	22
60-80	20	42
80-100	22	64

Here, $n = 64 \Rightarrow \dfrac{n}{2} = 32$

So, the median class is 60-80.

Thus, $f = 20$, $c.f. = 22$

$\therefore$ Median $= l + \left(\dfrac{\frac{n}{2} - c.f.}{f}\right) \times h$

$= 60 + \left(\dfrac{32-22}{20}\right) \times 20 = 60 + 10 = 70$

11. We know that tangents from an exterior point to a circle are equal in length.

$\therefore$ $AQ = AR$ ...(i), $BP = BQ$ ...(ii), $CP = CR$ ...(iii)

Now, perimeter of ΔABC

$= AB + BC + AC = AB + BP + CP + AC$

$= (AB + BQ) + (CR + AC)$ [Using (ii) and (iii)]

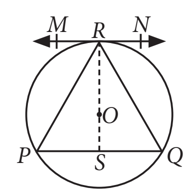
$= AQ + AR$

$= AQ + AQ$ [From (i)]

$= 2AQ$

Hence, $AQ = \dfrac{1}{2}$ (Perimeter of ΔABC)

OR



Let tangent MN is drawn at point R of the circle having centre O. Chord PQ is drawn parallel to the tangent line MN. Draw a perpendicular line OR on MN and extend it to intersect PQ at S.

$\therefore$ $RS \perp PQ$

In ΔRPS and ΔRQS,

$RS = RS$ [Common]

$\angle PSR = \angle QSR = 90°$ [$\because RS \perp PQ$]

$PS = SQ$ [$\because$ Perpendicular from centre to the chord bisects the chord]

$\therefore$ $\Delta RPS \cong \Delta RQS$ [By SAS congruence criterion]

$\Rightarrow$ $PR = QR$ [By CPCT]

We know that, if two chords are equal, then their corresponding arcs are also equal. Hence, R bisects the arc PRQ.

12. Let production in 1^{st} year be a units and increase in production (every year) be d units.

$\because$ Increase in production is constant.

$\therefore$ Unit produced every year forms an A.P.

Now, $a_3 = 6000 \Rightarrow a + 2d = 6000$

$\Rightarrow$ $a = 6000 - 2d$ ...(i)

and $a_7 = 7000 \Rightarrow a + 6d = 7000$

$\Rightarrow$ $(6000 - 2d) + 6d = 7000$ [Using (i)]

$\Rightarrow$ $4d = 1000 \Rightarrow d = 250$

Putting $d = 250$ in (i), we get

$a = 6000 - 2 \times 250 = 5500$

(i) Production in first year = 5500

(ii) Production in fifth year, $a_5 = a + 4d$

$= 5500 + 4 \times 250 = 6500$

(iii) Production in sixth year, $a_6 = a + 5d$

$= 5500 + 5 \times 250 = 6750$

13. (i) Let r be the radius of a small spherical ball.

Surface area of a spherical ball = 887.04 cm^2

$\Rightarrow \quad 4\pi r^2 = 887.04$

$\Rightarrow \quad r = 8.4$ cm

(ii) Radius of a cylindrical coin $= \frac{1}{4} \times 8.4 = 2.1$ cm

and height of a cylindrical coin = 4 cm

Now, curved surface area of a coin = $2\pi rh$

$= 2 \times \frac{22}{7} \times 2.1 \times 4 = 52.8 \text{ cm}^2$

14. (i) In the right ΔADQ, we have

$\sin 30° = \frac{DQ}{AD} \Rightarrow \frac{1}{2} = \frac{DQ}{24}$

$\Rightarrow \quad DQ = 12$ m

Thus, distance of paraglider from the ground is 12 m.

(ii) We have, $PQ = BC = 6$ m

Now, as $DQ = 12$ m

$\therefore \quad DP = DQ - PQ = 12 - 6 = 6$ m

In right ΔBDP, we have

$\sin 45° = \frac{DP}{BD} \Rightarrow \frac{1}{\sqrt{2}} = \frac{6}{BD}$

$\Rightarrow \quad BD = 6\sqrt{2}$ m

Thus, the distance of paraglider from the girl (Riddhima) is $6\sqrt{2}$ m.